

Low temperature resistance of quasi one-dimensional wires

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Abstract : The problem of the resistance of quasi one-dimensional (1D) wires is studied at temperatures low enough to preclude phonons. Localization becomes conspicuous at such temperatures. We have considered the inelastic scattering due to two level systems or tunneling states, which is dominant at low temperatures. The present study demonstrates that 1D localization is very sensitive to the frequency of inelastic scattering events. This calls for new experiments to investigate certain finer aspects of the 1D transport. The present study is also expected to be relevant in connection with the current discussion on the saturation of the intrinsic decoherence time at ultra low temperatures in mesoscopic systems.

Keywords : Localization, e-e interactions, two-level system

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1. Introduction

Classically, a thin wire of length L obeys Ohms law and the resistance R is expected to decrease with the lowering of the temperature. But the resistance of wires made of non-crystalline materials is found to depend differently on temperature, especially when the temperature is very low. Initially, there is a concomitant decrease of R with temperature but below a certain temperature an increase of R is observed [1]. Several theoretical approaches have been put forth, though not fully conclusive, to explain the microscopic origin of the increase of resistance which becomes spectacular below 12K [2,3]. Also, the scaling theory and the diffusive nature of the electrons' motion in the presence of disorder suggest that the resistance should be either a logarithmic or exponential function of length depending on whether the system is 2D or 1D [4].

One of the theoretical approaches was due to Thouless [2] and was based on the localization theory of the electrons according to which the extent to which the electron is localized depends on the strength of the disorder and the dimensionality of the system. Thouless wanted to know whether the result of complete localization in one dimension would also hold good for quasi one-dimensional wires which are actually realised in the laboratory. Considering the diffusive behaviour of the electron wave packet, he could arrive at an expression for the increment in resistance due to localization believing that the

reduction in temperature will enhance the inelastic collision time τ_i with phonons thereby enabling the electrons to travel over distances of the order of localization length. This contention was however not borne out by experiments [5-8] which showed considerably less effect of localization. This could imply that the inelastic scattering time was lesser than that expected by Thouless. Black *et al.* [9] at this stage realised the importance of inelastic scattering from tunneling states (TS) or two-level systems [10] and suggested that it reduces the scattering time τ_i and therefore also the effect of localization considerably. Although the microscopic origin of the tunneling states is not known as yet, the hypothesis has helped in explaining many of the low temperature anomalies in glasses.

The other theoretical approach treating the disordered Fermi liquid problem by perturbation theory to the lowest order in interaction strength developed by Altshuler *et al* [3] also gave the same dependence for the increment in resistance on A and T as was seen in Thouless's theory. But both the theories had a discrepancy with regard to the dependence on ρ_B , the bulk resistivity. Though there were experimental results which favoured either of these theories [5-8, 11] and some which favoured the simultaneous presence of both of them [12], the localization theory seems to have an edge over the interaction theory in so far as the ρ_B dependence is concerned [13, 14].

We have arrived at an expression for the increment in R due to localization which gives the correct dependence on A , T and ρ_B [14] taking into account the fact that at very low temperature

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when other typical sources of inelastic scattering diminishes the contribution from TSs persists and becomes the dominant source of scattering.

2. Localization correction to resistance

We have considered the movement of an electron as an elastically diffusing narrow wavepacket made up of broad localized states which diffuses along the length of the wire until an inelastic event like phonon or TSs scatters it to another localized state wherein it starts again from the scratch. The concept of disorder is implicitly embedded in the frequency or probability of the inelastic scattering from the TS. We have expressed the range of diffusion of the wave packet in terms of step lengths $l_r = ra_0$, $r = 1, 2, 3, \dots$; and a_0 is the interatomic distance and $L_r = rl$ with $r = 0, 1, 2, \dots$ and l is the localization length which in turn depends upon the temperature and the concentration of the TSs present in the system. The inelastic lifetime in the two regimes are taken as $\tau_{<}$ and $\tau_{>}$.

In a quasi 1D wire of length L , l_r and L_r can have $s(l_r)$ and $S(L_r)$ number distributions which peak at different values of segment length. Using the results of the scaling theory, the net dimensionless resistance scaled in $\pi \hbar / e^2$, is just an incoherent sum with segments contributing as resistances in series :

$$R = \sum_{r=1}^N s(l_r)(e^{l_r/l} - 1) + \sum_{r=1}^n S(L_r)(e^{L_r/l} - 1). \quad (1)$$

At higher T , metallic behaviour sets in and the incremental resistance ratio is

$$\frac{\Delta R}{R_0} = \sum_{r=1}^N p_r \left[\left(\frac{e^{l_r/l} - 1}{l_r/l} \right) - 1 \right] + \sum_{r=0}^n P_r \left[\left(\frac{e^{L_r/l} - 1}{L_r/l} \right) - 1 \right], \quad (2)$$

where we have defined $s(l_r)l_r / L \equiv p_r$ and $S(L_r)L_r / L \equiv P_r$; as the fractional contributions of the two path lengths to the total length, with $\sum_{r=1}^N p_r + \sum_{r=0}^n P_r \equiv (1 - \alpha) + \alpha = 1$. On an average, we will always take $L_r < l$ since the probability for undergoing variable-range-hopping is exponentially smaller than that for finding an inelastic scatterer. Each term in (5) dominates separately in one of the regimes described above.

3. Short and long step regimes

The localization correction from the packets that are able to diffuse over short distances well within the localization length l due to frequent scatterings from the TSs is

$$\frac{\Delta R_{<}}{R_0} \approx (1 - \alpha) \frac{\langle l_r \rangle}{2l} \approx (1 - \alpha) \frac{\rho_B}{A} \frac{\sqrt{D\tau_{<}}}{\sqrt{2}(\pi \hbar / e^2)}. \quad (3)$$

This result is the same as that of Thouless but is derived here more directly from the exponential length dependent of R . $\tau_{<}$ can be small either due to frequent scatterings from phonons

or due to scatterings from TSs. If the inelastic events are less frequent such that the wavepackets can diffuse over distances of the order of l , then $\frac{\Delta R_{>}}{R_0} \approx \frac{\alpha}{2} \langle r \rangle$, where $\langle r \rangle$ is the coarse grained average.

The frequency of inelastic events has been taken care of by introducing the probability $P_{>}$ for a packet to take a step of size l , the trajectory of which is represented as randomly arranged bars $n_{>}$ in number and a probability $P_{<}$ of not diffusing represented by vertical partitions $n_{<}$ in number such that $P_{>} + P_{<} = 1$ and $n_{>} + n_{<} = n$. The coarse grained average can then be calculated and the most probable values $\bar{n}_{<}$ and $\bar{n}_{>}$ are found by maximizing the binomial distribution using Stirlings approximation and we get $\langle r \rangle \approx P_{>} / P_{<}$.

The probabilities $P_{>}$ and $P_{<}$ defined in the strictly 1D model is related to the physical decay and diffusion rates in the quasi 1D wire. In a thin wire, the diffusing packets can be taken as uniform in the transverse direction while diffusing "one-dimensionally" at a rate D/l^2 along the wire and the 1D diffusion probability $P_{>}$ then depends directly on it. Since there are A/a_0^2 chains of states across the wire, the decay in any one of them will cause incoherence in the overall combination and hence the rate per chain is to be related to the 1D probability $P_{<}$. Then, we get $P_{>} / P_{<} = (2D/l^2) / (a_0^2 A \tau_{>})$ and finally

$$\Delta R_{>} / R_0 = \alpha \rho_B^2 D \tau_{>} / \{ A a_0^2 (\pi \hbar / e^2)^2 \}. \quad (4)$$

The energy width of each localized state which make up the packet will affect the intrinsic energy width of the packet and consequently its intrinsic incoherence time. Since the details of how a packet is constructed is not known, we have considered a typical diffusion path length and worked backwards to relate $\tau_{>}(\tau_l)$ to \hbar / τ_l .

A free wavepacket narrows in energy space as it travels such that $\Delta E \geq \hbar K / \langle x(t) \rangle$. We assume that for an elastically diffusing wave packet also the same relation holds. Since the localized states decay in τ_l , the intrinsic energy width of the packet should be $\hbar / \tau_{>} \geq \hbar K / \sqrt{2D\tau_l}$. The constant K is estimated by considering a narrow packet of size $\sim a_0$ and intrinsic width \hbar / τ_0 which does not narrow in its short life time τ_0 (i.e.) $\hbar / \tau_0 \geq \hbar K / \sqrt{2D\tau_0}$ and we get $\tau_{>} \approx \sqrt{\tau_0 \tau_l}$. Therefore

$$\frac{\Delta R_{>}}{R_0} = \alpha \left| \frac{\rho_B^2}{a_0^2 A} D \sqrt{\tau_l \tau_0} / (\pi \hbar / e^2)^2 \right|. \quad (5)$$

Thus, we get $\frac{\Delta R_{>}}{R_0} \sim \rho_B A^{-1} T^{-1/2}$ in agreement with the experimental results. Using the parameters of [8], one finds $\Delta R_{>} / R_0 = 4 \times 10^{-2}$ as compared to the experimental value $\sim 2 \times 10^{-2}$. Thouless' result was found to be anomalously large [8, 15].

4. Saturation of decoherence time

Contrary to that expected from theory the temperature dependence of the phase decoherence time in mesoscopic disordered systems is found to saturate at a certain value decided by the system characteristics [16]. Though some authors (see Mohanty *et al.* in [16]) have attributed the saturation to be due to the zero point fluctuations of either the electrons or the impurities, these views have been refuted. The persistence of TSs at ultralow temperatures and the change of nature of tunneling from coherent to incoherent with decreasing temperature [17] motivates us to propose that they might be responsible for the observed temperature dependence of the decoherence time.

5. Discussion

The electrical resistance of quasi-1D systems is seen to be sensitive to the frequency of inelastic scattering events. New experiments are needed under controlled conditions of disorder and temperature to study in detail the roles of inelastic scatterings from TSs and phonons separately in quasi- 1D systems. This should give more insight into the microscopic details of electron diffusion in the backdrop of localization. Further, the role of TSs in limiting the phase incoherence time needs to be investigated in detail.

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